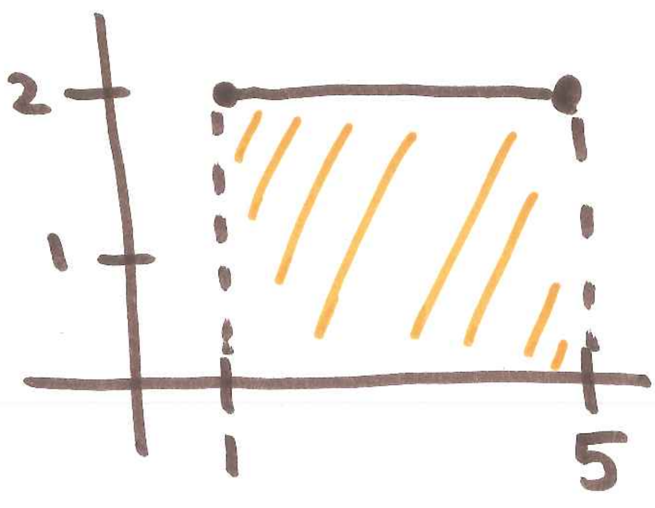
Chapter 8 - Day 1

Ex: find the area of the region bounded above by $f(x)=2$, below by the x-axis, on the left by $x=1$ and the right by $x=5$.



$A = l \cdot h$

$= 4 \cdot 2$

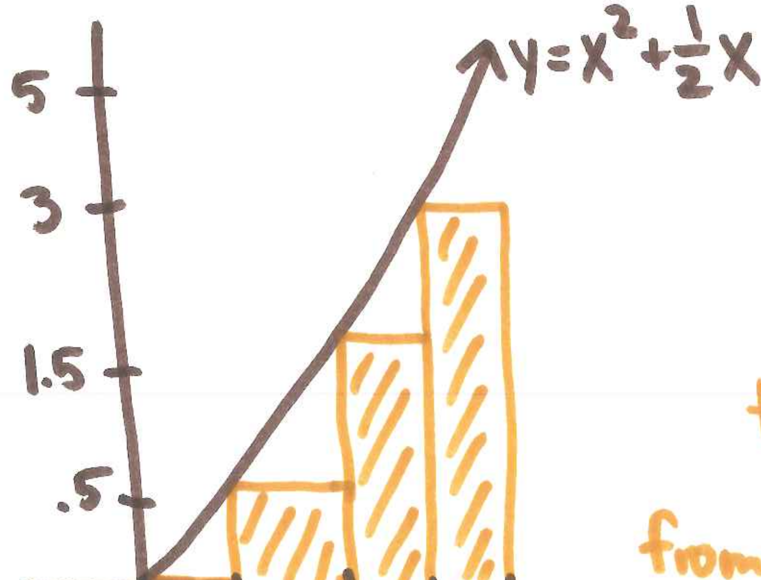
$= \boxed{8 \text{ units}^2}$

Ex: A car is traveling at a constant velocity of 55 mph. How far does the car travel between 11 am and 1 pm?

$$d = r \cdot t$$
$$= 55(2)$$
$$= \boxed{110 \text{ miles}}$$

Ex: Estimate the area under the graph of $y = x^2 + \frac{1}{2}x$ for $x \in [0,2]$ two ways.

a) Subdivide $[0,2]$ into 4 equal subintervals and use the left endpoint as a "sample point".



Now, we have 4 rectangles to find the area of.

from $0 \to \frac{1}{2}$ $\ell = \frac{1}{2}$ $h=0$ $A=0$

from $\frac{1}{2} \to 1$ $\ell = \frac{1}{2}$ $h=.5$ $A = 1/4$

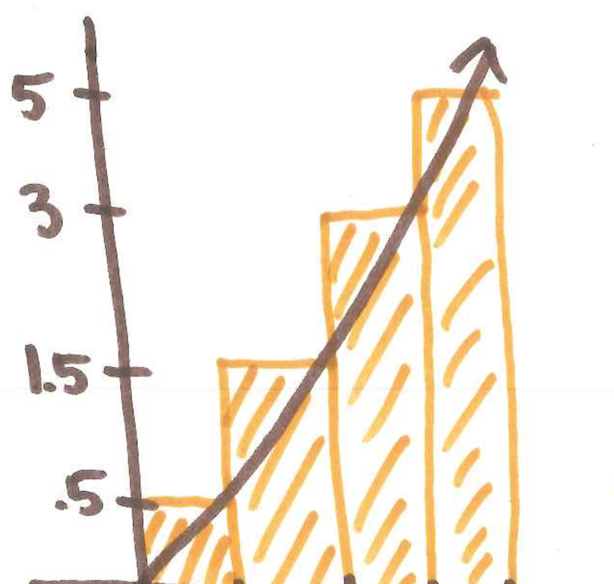
from $1 \to 1\frac{1}{2}$ $\ell = \frac{1}{2}$ $h=1.5$ $A = 3/4$

from $1\frac{1}{2} \to 2$ $\ell = \frac{1}{2}$ $h=3$ $A = 3/2$

Approximate area under the curve

is $= 0 + 1/4 + 3/4 + 3/2 = \boxed{2\tfrac{1}{2} \text{ units}^2}$

b) Subdivide $[0,2]$ into 4 equal subintervals and use the right endpoint as "sample point".



Again, let's find the area of the 4 rectangles.

from $0 \to \frac{1}{2}$ $\ell = \frac{1}{2}$ $h = .5$
$A = 1/4$

from $\frac{1}{2} \to 1$ $\ell = \frac{1}{2}$ $h = 1.5$
$A = 3/4$

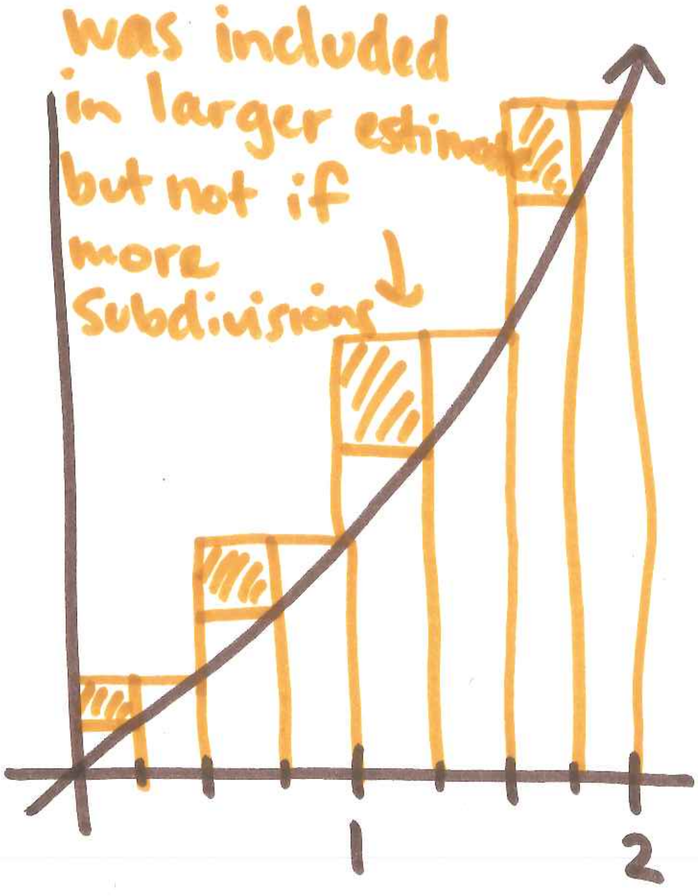
from $1 \to 1\frac{1}{2}$ $\ell = \frac{1}{2}$ $h = 3$
$A = 3/2$

from $1\frac{1}{2} \to 2$ $\ell = \frac{1}{2}$ $h = 5$
$A = 5/2$

Approximate area under curve is

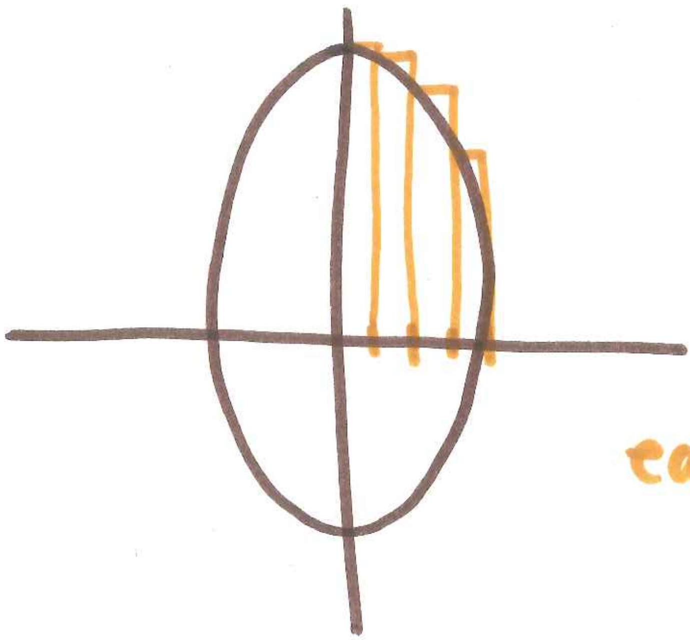
$= \frac{1}{4} + \frac{3}{4} + \frac{3}{2} + \frac{5}{2} = \boxed{5 \text{ units}^2}$

What's the difference between these 2 estimates? $5 - 2\frac{1}{2} = \boxed{2\frac{1}{2} \text{ units}^2}$



If we choose more subdivisions, we'd get a better estimate because there is less variation on smaller intervals.

Ex: Estimate the area of the ellipse given by $4x^2 + y^2 = 49$.



consider 1st quadrant
$x \in [0, 3.5]$

Use 4 subdivisions and left endpoints.

each subdivision is $\frac{3.5}{4} = .875$ of a unit

$$y^2 = 49 - 4x^2$$
$$f(x) = y = \sqrt{49 - 4x^2}$$

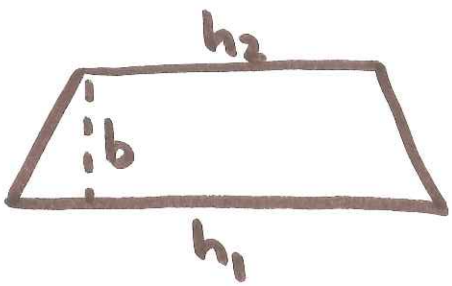
$$\text{Area} = .875(f(0)) + .875(f(.875)) + .875(f(1.75)) + .875(f(2.625))$$
$$= .875(7) + .875(6.778) + .875(6.062) + .875(4.630)$$
$$= 21.411$$

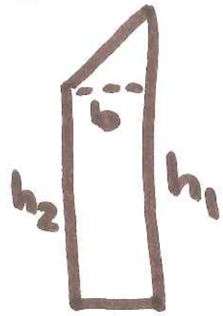
This was only 1/4 of the ellipse!

$$21.411 \times 4 = \boxed{85.644 \text{ units}^2}$$

Can we do a better estimate?

Yes!. Use trapezoids.



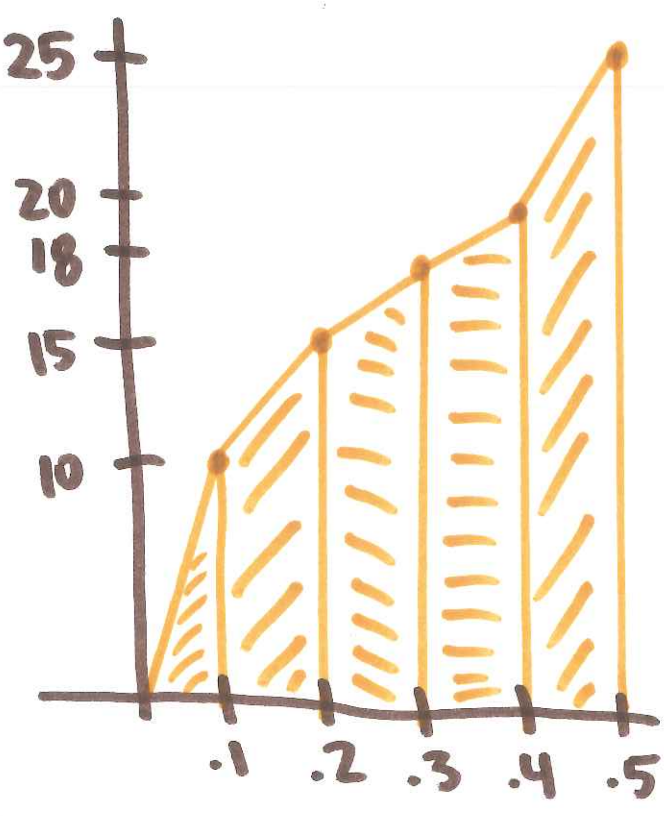


$$\text{area of a trapezoid} = \frac{(h_1+h_2)\cdot b}{2}$$

Ex: A train is traveling along a track, velocity varies, but is measured every 1/10 hr. For the first half hour, measurements are listed

time	0	.1	.2	.3	.4	.5
velocity	0	10	15	18	20	25

Compute the total distance traveled by the train.



$$\text{distance} = \frac{(0+10)(.1)}{2} + \frac{(10+15)(.1)}{2} + \frac{(15+18)(.1)}{2} + \frac{(18+20)(.1)}{2} + \frac{(20+25)(.1)}{2}$$

$$= .5 + 1.25 + 1.65 + 1.9 + 2.25$$

$$= \boxed{7.55 \text{ miles}}$$